

# Trade Selection with Supervised Learning and Optimal Coordinate Ascent (OCA)

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To appear in ECML PKDD workshop MIDAS 2020

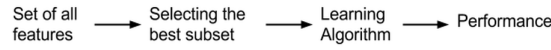
**Abstract.** Can we dynamically extract some information and strong relationship between some financial features in order to select some financial trades over time? Despite the advent of representation learning and end-to-end approaches, mainly through deep learning, feature selection remains a key point in many machine learning scenarios. This paper introduces a new theoretically motivated method for feature selection. The approach that fits within the family of embedded methods, casts the feature selection conundrum as a coordinate ascent optimization with variables dependencies materialized by block variables. Thanks to a limited number of iterations, it proves efficiency for gradient boosting methods, implemented with XGBoost. In case of convex and smooth functions, we are able to prove that the convergence rate is polynomial in terms of the dimension of the full features set. We provide comparisons with state of the art methods, Recursive Feature Elimination and Binary Coordinate Ascent and show that this method is competitive when selecting some financial trades.

## 1 Introduction

Feature selection is also known as variable or attribute selection. This method concerns the selection of a subset of relevant attributes in our data that are most relevant to our predictive modeling problem. It has been an active and fruitful field of research and development for decades in statistical learning. It has proven to be effective and useful in both theory and practice for many reasons: enhanced learning efficiency and increasing predictive accuracy (see [21]), model simplification to ease its interpretation and improve performance (see [1], [18] and [4]), shorter training time (see [21]), curse of dimensionality avoidance, enhanced generalization with reduced overfitting, and implied variance reduction. Both [14] and [13] are nice references to get an overview of various methods to tackle feature selections. The approaches followed vary; briefly speaking, the methods can be sorted into three main categories: Filter method, Wrapper methods and Embedded methods. We develop these three categories in the following section.

### 1.1 Filter methods

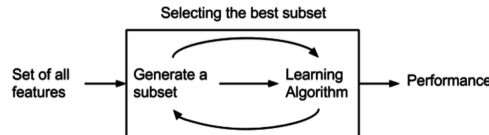
Filter type methods select variables despite the model. These methods suppress the least interesting variables using ranking techniques as a criteria to select the variables. Once the ranking is done, a threshold is determined in order to select features with rank above it. These methods are very effective in terms of computation time and robust to overfitting. One famous Filter method approach is the algorithm developed in [15] in 1992, called *Relief*, for application to binary classification problems. By construction, Filter methods may select redundant variables as they do not consider the relationships between variables. One of the most used criteria for Filter methods is the Pearson correlation coefficient, which is simply the ratio between the covariance and the square root of the two variances:  $\text{Cov}(x_i, y) / \sqrt{\text{Var}(x_i) \text{Var}(y)}$  with  $x_i$  the  $i^{\text{th}}$  feature in the model and  $y$  the label associated. It is well known that this correlation ranking can only detect linear dependencies between features and the target label. The Filter method procedure is summarized in figure 1.



**Fig. 1.** Filter method: it consists in 4 specific steps. Arrows emphasize that these steps are done in chronological order.

### 1.2 Wrapper methods

Wrapper methods allow detecting possible interactions between variables by evaluating subsets of them. In Wrapper methods, a model must be trained to test any subsequent feature subset. Consequently, these methods are iterative and computationally expensive. However, these methods can identify the best performing features set for that specific modeling algorithm. These methods are explained in figure 2. Some known examples of Wrapper methods are forward



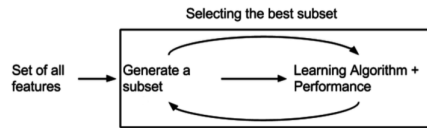
**Fig. 2.** Wrapper method: like filter method, it consists in 4 different steps but there are iterations between steps 2 and 3 emphasized by the rectangle until each feature subset combination is computed.

and backward feature selection methods. These methods are presented in the

work of [17]. The backward elimination starts with all features and progressively remove them. At the opposite, the forward selection starts with an empty set and progressively add them. If we have  $n$  features, we need to train  $n$  classifiers for the first step, then  $n - 1$  classifiers for the second step and so on. We then have  $n(n + 1)/2$  training steps for both methods. However, forward selection starts with small features subsets so it can be computationally cheaper if the stopping condition is satisfied early. An adaptative forward-backward algorithm is proposed in [29]. One of the state of the art Wrapper method is Recursive Feature Elimination (RFE) (see for instance [20] for more details). It first fits a model and removes features until a pre-determined number of features. Features are ranked through an external model that assigns weights to each features and RFE recursively eliminates features with the least weight at each iteration. One of the main limitation to RFE is that it requires the number of features to keep. This is hard to guess a priori and one may need to iterate much more than the desired number of feature to find an optimal feature set.

### 1.3 Embedded methods

Embedded methods perform feature selection as a part of the modeling algorithm's execution. Many hybrid methods are developed to combine the advantages of Wrapper and Filter methods. These methods combine learning and feature selection through a prior or a posterior regularization. The different methods of regularization are given in [10]. The use of L1 regularization is introduced in [24] and a comparison between L1 and L2 regularization is explained in [23]. Besides, an extension to non-linear spaces is given in [2], using Multiple Kernel Learning (MLKL). These methods are summarized in figure 3. Very recently



**Fig. 3.** Embedded method: as opposed to filter and wrapper methods, there are only 3 steps as the learning and performance steps are combined into a single step. Like for the wrapper method, we iterate between step 2 and 3 until we find the best features subset among all combinations.

there are also many methods brought by the field of machine learning interpretability, like Shapley values calculation[?] and local surrogates (LIME) [?] that can be used for features selection being run on the train set.

## 2 Framework

### 2.1 Block variables

[8] deals with the choice of the number of block to consider in a feature selection problem. We propose an answer to the open problem discussed in [3] about solving an unconstrained minimization using a projection method in which each iteration consists of performing a gradient projection step with respect to a certain block taken in a cyclic order or using an alternating minimization method using only two blocks. Indeed, we decide to split the data into  $n \geq 2$  blocks  $\left( [B^1] \dots [B^n] \right)$  and instead of simply performing a gradient ascent method on a single block at each iteration, we first initialize our method by finding the best sub-block data set in terms of prediction score for our supervised learning problem and then apply a gradient ascent method on each single block at every iteration (for more details, see section 3).

### 2.2 Result of convergence

In order to motivate our method that relies on coordinate ascent, we recall some theoretical results about the convergence of coordinate ascent optimization. The theory is well understood for the convex case (see [27]). The non convex case without gradient which is our example is however much harder as we have local minima issue and mathematical assumptions too weak to be able to prove convergence. However, convergence results under strong convex conditions provide some hint about the efficiency of this method and its convergence rate that is linear. The proof is given by [22]. In order to have some meaningful result, we need to make some necessary assumptions for our function  $f$  to be minimized. We assume that we have a real value  $n$ -dimensional function  $f : \mathbb{R}^n \rightarrow \mathbb{R}$ . In this section, we stick to the traditional presentation and examine minimization to make proof reading easier. We examine the following optimization program:

$$\min_{x \in \mathbb{R}^n} f(x) \tag{1}$$

The notation and assumptions used for the two following proposition are given by [22].

**Proposition 1.** *Under assumption given by [22], coordinate ascent optimization converges to the global minimum  $f^*$  at a linear rate proportional to  $2nL_{\max}R_0^2$ , that is*

$$\mathbb{E}[f(x_k)] - f^* \leq \frac{2nL_{\max}R_0^2}{k} \tag{2}$$

*Proof.* The proof is given by [22].

We have an additional control of our convergence rate which is given by the following proposition.

**Proposition 2.** *Under assumption [22], and for  $\sigma > 0$ , coordinate ascent optimization error is controlled by the following inequality*

$$\mathbb{E}[f(x_k)] - f^* \leq \left(1 - \frac{\sigma}{nL_{\max}}\right)^k (f(x_0) - f^*) \quad (3)$$

*Proof.* The proof is given by [22].

### 3 Method developed

In many applications, we can regroup features among families. We call these features block variables. Typical example is to regroup variables that are observations of some physical quantity but at a different time (like the speed of the wind measure at different hours for some energy prediction problem, the price of a stock in an algorithmic trading strategy for financial markets, the temperature or heart beat of a patient at different time, ...). We denote from now  $\mathcal{M}_{(n,p)}$  the space of matrices of  $n$  rows and  $p$  columns. Supposing that we have  $J$  rows of data, we can formally regroup our variables into two sets:

- the first set encompasses  $[B^1] \dots [B^n]$ . These are block variables of different length  $L_i$ . Mathematically, the sub-block variables are denoted by  $B_j^i$  with  $(B_j^i)_{\substack{i \in 1 \dots n \\ j \in 1 \dots L_i}}$  taking value in  $\mathbb{R}^J$ .
- the second set is denoted  $[S]$  and is a block of  $p$  single variables.

Graphically, our variables looks like that:

$$\left( \begin{array}{c|c|c} \overbrace{B^1} & \overbrace{B^n} & \overbrace{S} \\ \hline \begin{array}{ccc} B_1^1 & \dots & B_{L_1}^1 \\ \bullet & \dots & \bullet \\ \vdots & & \vdots \\ \bullet & \dots & \bullet \end{array} & \begin{array}{ccc} B_1^n & \dots & B_{L_n}^n \\ \bullet & \dots & \bullet \\ \vdots & & \vdots \\ \bullet & \dots & \bullet \end{array} & \begin{array}{ccc} S_1 & \dots & S_p \\ \bullet & \dots & \bullet \\ \vdots & & \vdots \\ \bullet & \dots & \bullet \end{array} \end{array} \right)$$

Thus, we have  $N$  variables split between block variables and single variables, hence  $N = N_B + p$  with  $N_B = \sum_{i=1}^n L_i$ .

Our algorithm works as follows. We first fit our classification model to find a ranking of features importance. The performance is computed using the Gini index for each variable. Gini Index, or Gini impurity, computes the probability of a specific feature that is classified incorrectly when selected randomly. It can be mathematically written as:  $Gini = 1 - \sum_{i=1}^n P_i^2$  with  $P_i$  the probability of a feature being classified for some distinct class. We then keep the first  $k$  best ranked features for each blocks  $B^1 \dots B^n$  in order to find the best initial guess for our coordinate ascent algorithm. Notice that the set of unique variables is not modified during the first step of the procedure. The objective function is the number of correctly classified samples at each iteration. It can be expressed as followed:

$$accuracy = \frac{TP + TN}{\text{number of examples}} \quad (4)$$

with True Positives (TP) denoting the number of positive examples labeled as positive and True Negatives (TN) the number of negative examples labeled as negative.

*Remark 1.* Notice that the number of examples (denoted by  $n$  in this remark) respects the following relation :  $n = TP + TN + FP + FN$  with False Positives (FP) denoting the number of negative examples labeled as positive and False Negatives (FN) the number of positive examples labeled as negative.

We then enter the main loop of the algorithm. Starting with the vector of  $(k, \dots, k, \mathbb{1}_p^T)$  as the initial guess for our algorithm, we perform our coordinate ascent optimization in order to find the set with optimal score and the minimum number of features. The coordinate ascent loop stops whenever we either reach the maximum number of iterations or the current optimal solution has not moved so far between two steps.

Taking the previous notation, we start with the initial block data set  $\left( [B_1^1 \dots B_{L_1}^1] \dots [B_1^n \dots B_{L_n}^n], [S] \right)$  and reduce it to the new data set  $\left( [B^{1,\diamond}] \dots [B^{n,\diamond}], [S] \right)$ , with  $B^{i,\diamond} \in \mathcal{M}_{(J,k)} \quad \forall i \in 1 \dots n$ . We can first apply a coordinate ascent optimization on each single block and get a new data set structure without considering blocks. We secondly apply a binary coordinate ascent optimization on the remain data set in order to get the final data set  $\left( [B^{1,*}] \dots [B^{n,*}], [S^*] \right)$ . We summarize the algorithm in the pseudo code 1. To control early stopping, we use precision variables denoted by  $\varepsilon_1$  and  $\varepsilon_2$ , and two maximum iterations  $\text{Iteration max}_1$  and  $\text{Iteration max}_2$  that are initialized before starting the algorithm. We also denote  $\text{Score}(k_1, \dots, k_n, \mathbb{1}_p)$  to be the accuracy score of our classifier with each  $B_i$  block of variables retaining  $k_i$  best variables and with single variable all retained.

*Remark 2.* The originality of this coordinate ascent optimization is to regroup variable by blocks, hence it reduces the number of iterations compared to Binary Coordinate Ascent (BCA) as presented in [28]. The stopping condition can be changed to accommodate for other stopping conditions.

*Remark 3.* The specificity of our method is to keep the  $j$  best representative features for each features class, as opposed to other methods that only select one representative feature from each group, ignoring the strong similarities between each feature of a given variable block. This takes in particular the opposite view of feature Selection with Ensembles, Artificial Variables, and Redundancy Elimination as developed in [25].

## 4 Numerical Results

### 4.1 Data set

We carry out our experiment on real finance data set. In financial markets, algorithmic trading has become more and more standard over the last few years.

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**Algorithm 1** OCA algorithm :

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**J Best optimization**

We retrieve features importance from a fitted model

We find the index  $k^*$  that gives the best score for variables block of same size  $k$ :

$$k^* \in \underset{k \in \mathbb{R}^{L_{\min}}}{\operatorname{argmax}} \operatorname{Score}(k, \dots, k, \mathbb{1}_p)$$

Initial guess :  $x^0 = (k^*, \dots, k^*, \mathbb{1}_p)$ **while**  $|\operatorname{Score}(x^i) - \operatorname{Score}(x^{i-1})| \geq \varepsilon_1$  and  $i \leq \operatorname{Iteration}_{\max_1}$  **do**

$$x_1^i \in \underset{j \in \mathbb{R}^{L_1}}{\operatorname{argmax}} \operatorname{Score}(j, x_2^{i-1}, x_3^{i-1}, \dots, x_n^{i-1}, \mathbb{1}_p)$$

...

$$x_n^i \in \underset{j \in \mathbb{R}^{L_n}}{\operatorname{argmax}} \operatorname{Score}(x_1^i, x_2^i, x_3^i, \dots, j, \mathbb{1}_p)$$

 $i += 1$ **end while****Full coordinate ascent optimization**Use previous solutions:  $X^* = (x_1^i, \dots, x_n^i, \mathbb{1}_p)$ 

$$Y^* = \operatorname{Score}(X^*)$$

**while**  $|Y - Y^*| \geq \varepsilon_2$  and iteration  $\leq \operatorname{Iteration}_{\max_2}$  **do****for**  $i=1 \dots N$  **do**

$$X = X^*$$

$$X_i = \text{not}(X_i^*)$$

**if**  $\operatorname{Score}(X) \geq \operatorname{Score}(X^*)$  **then**

$$X^* = X$$

**end if****end for**

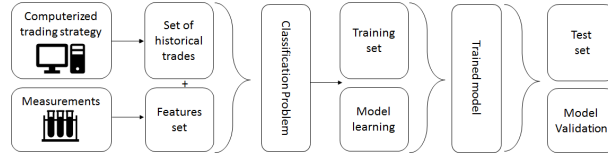
$$Y = \operatorname{Score}(X^*)$$

iteration  $+= 1$ **end while**Return  $X^*, Y^*$ 

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The rise of the machine has been particularly significant in liquid and electronic markets such as foreign exchange and futures markets reaching between 60 to 80 percent of total traded volume (see for instance [6], [12] or [5] for more details on the various markets). These strategies are even more concentrated whenever there are very fast market moves as reported in [16]. A trading strategy is usually defined with some signal that generates a trading entry. But once we are in position, then main following point is the trading exit strategy. There are multiple method to handle efficient exits, ranging from fixed target and stop loss, to dynamic target and stop loss. Indeed, to enforce success and crystallize gain or limit loss, a common practice is to associate to the strategy a profit target and stop loss as described in various papers ([19], [11], [9], or [26]). We use our algorithm to do a supervised classification according to some a priori features. We are given 1500 trades over a ten years history with 135 features that can be classified into five blocks of twenty variables, one block of thirty variables and five single variables. Each block can be interpreted as a temporally relationship between some technical analysis indicators or price history. We know for each trade whether it is a

‘good’ or ‘bad’ trade (thanks to the gain and loss, denoted by PnL, engendered by the trade). The idea is to use the minimum number of features to classify a priori this data set. We use cross validation with 70% for the training set and 30% for the test sets. The procedure is summarized in figure 4. For full reproducibility, full data set and corresponding python code for this algorithm is available publicly on github: [https://github.com/aisquareconnect/features\\_selection\\_oca](https://github.com/aisquareconnect/features_selection_oca). We consider a gradient boosting framework by using XGBoost library, initially developed in [7], in the implementation of our algorithm. It is illuminating to



**Fig. 4.** Learning process for our trade selection challenge. We first use a proprietary trading strategy that generates some samples trades. We take various measures before the trades are executed to create a feature set. We combined these to create a supervised learning classification problem. Using xgboost method and OCA, we learn model parameters on a train set. We monitor overall performance of the trading strategy on a separate test set to validate scarce overfitting.

look at the histogram of gain and losses of our trades over our 10 years of history. We can observe two peaks corresponding to the profit target and stop loss level as shown in figure 5. It is much better to use the gain and loss curve in the native currency of the underlying instrument than to look at the consolidated currency of our trading strategies to avoid foreign exchange noise. We will consider two data sets of a ‘bad’ strategy, called strategy 1, and a ‘good’ one, called strategy 2; each strategy engendering trades among a fixed period of time.

The graph of the evolution of the gain and loss of the two strategies is given in figure 6.

*Remark 4.* The data set of the strategy 1 is quite balanced in terms of label (PnL engendered by the strategy) whereas it is not the case for the second one, we can’t thus apply the definition of the accuracy given in (4) because it would perform badly and it is recommended to use the definition of the balance accuracy, that is (taking the previous notation introduced in (4) and remark 1:

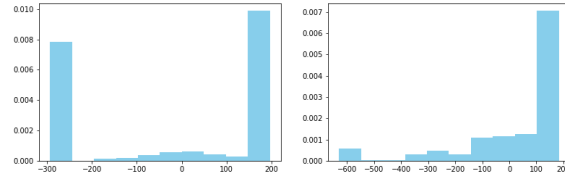
$$\text{Balance accuracy} = \frac{TPR + TNR}{2}$$

with the True Positive Rate (TPR) and the True Negative Rate (TNR) defined as :

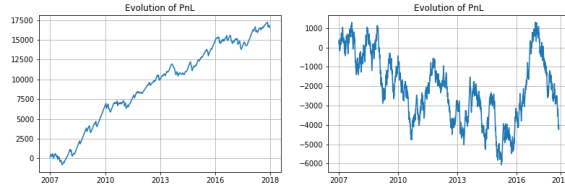
$$TPR = \frac{TP}{TP + FN}, \quad TNR = \frac{TN}{TN + FP}$$

True Positive Rate (TPR) concentrates on getting accurate results on the positive labels, regardless of false results. On the opposite, True Negative Rate





**Fig. 5.** Histogram of the PnL of the the two strategies in native currency. The left histogram corresponds to the strategy 1 and the right one to the second one. As the algorithm is performed on an underlying asset listed on an American financial market, the resulting strategy is denominated in USD. The strategy uses fixed stop loss and profit target level. This leads to two major columns corresponding for the left peak to trades that end in a loss when they hit the stop loss level and for the right peak to trades that exit with a profit as they reach the profit target.



**Fig. 6.** Evolution of the gain and loss of the two strategies.

(TNR) focuses on avoiding to incorrectly classify as a true label something that is a negative label. This is similar to type I and type II error in statistical tests.

## 4.2 Comparison

We first analyze the results given by the strategy 1. We compare our method to two other methods that are supposed to be state of the art for feature selections, namely RFE and BCA. Our new method achieves a score of 62.80 % with 16% of features used, to be compared to RFE that achieves 62.80 % with 19% of features used. BCA performs poorly with a highest score given by 62.19 % with 27.08% of features used. If we take in terms of efficiency criterium, the highest score with the less feature, our method is the most efficient among these three methods. In comparison, with the same number of features, namely 16.6%, RFE gets a score of 62.39 %. All these figures are summarized in the table 1.

## 5 Discussion

Compared to BCA our method reduces the number of iterations as it uses the fact that variables can be regrouped into categories or classes. The number

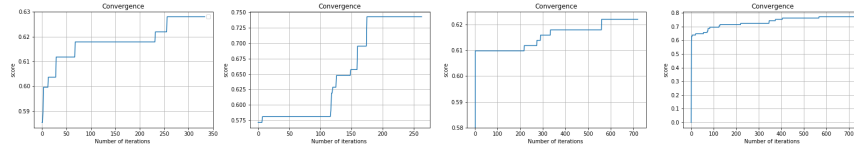
**Table 1.** Method comparison for the strategy 1: for each row, we provide in red the best(s) (hottest) method(s) and in blue the worst (coldest) method, while intermediate methods are in orange. We can notice that OCA achieves the higher score with the minimum feature sets. For the same cardinal of features set, RFE performs worst or equally, if we want the same performance for RFE, we need to have a larger feature set. BCA is the worst method both in terms of score and minimum feature set.

Method	% of features	Score (in %)
OCA using 24 features	16.6	62.8
RFE using 24 features	16.6	62.39
BCA using 39 features	27.08	62.19
RFE using 28 features	19.4	62.8

**Table 2.** Method Comparison for the strategy 2:

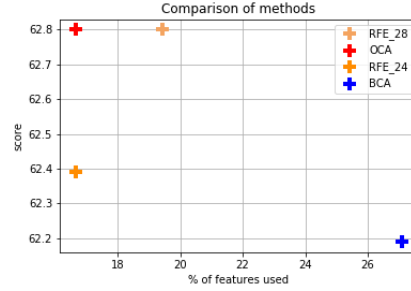
Method	% of features	Score (in %)
OCA using 10 features	6.75	74.28
RFE using 10 features	6.75	50.47
BCA using 52 features	35.13	77.14

of iterations for both OCA and BCA methods is provided below in figure 7. Our method requires only 350 iterations steps to converge as opposed to BCA that needs up to 700 iterations steps as it considers blindly variables ignoring similarities between them.

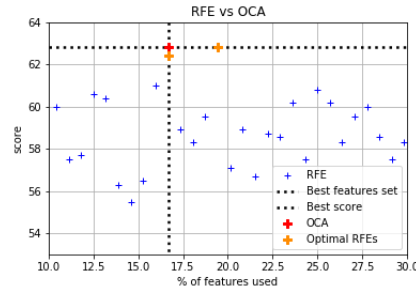


**Fig. 7.** Iterations steps up to convergence for OCA and BCA. OCA method is above while BCA is below. The figures on the left represent the strategy 1 and the strategy 2 is represented on the right. We see that OCA requires around 250 or 350 iteration steps to converge whereas BCA requires the double (around 700 iteration) steps to converge.

Graphically, we can determine the best candidates for the three methods listed in table 2 in figures 8, 9 and 10 for both strategies. We have taken the following color code. The hottest (or best performing) method is plotted in red, while the worst in blue. Average performing methods are plotted in orange. In order to compare finely OCA and RFE, we have plotted in figure 9 the result of RFE for used features set percentage from 10 to 30 percent. We can notice that for the same feature set as OCA, RFE has a lower score and equally that to get the same score as OCA, RFE needs a large features set.



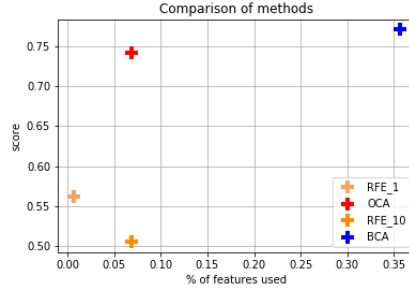
**Fig. 8.** Comparison between the 3 methods for strategy 1. To qualify the best method, it should be in the upper left corner. The desirable feature is to have as little features as possible and the highest score. We can see that the red cross that represents OCA is the best. The color code has been designed to ease readability. Red is the best, orange is a slightly lower performance while blue is the worst.



**Fig. 9.** Comparison between OCA and RFE for strategy 1: For RFE, we provide the score for various features set in blue. The two best RFE performers points are the orange cross marker points that are precisely the one listed in table 1. The red cross marker point represents OCA. It achieves the best efficiency as it has the highest score and the smallest feature set for this score.

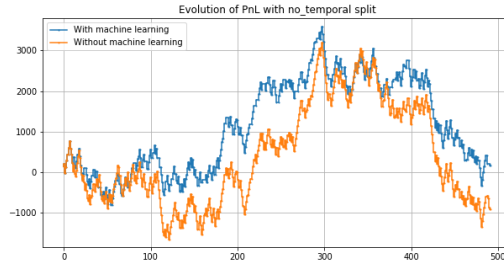
### 5.1 Reduced overfitting

We look at the final goal which is to compare the trading strategy with and without machine learning. A standard way in machine learning is to split our data set between a randomized training and test set. We keep one third of our data for testing to spot any potential overfitting. If we use the standard and somehow naive way to take randomly one third of the data for our test set, we break the time dependency of our data. This has two consequences. We use in our training set some data that are after our test sets which is not realistic compared to real life. We also neglect any regime change in our data by mixing data that are not from the same period of time. However, we can do the test on this mainstream approach and compare the trading strategy with and without machine learning filtering. This is provided in figure 11. The orange (respectively



**Fig. 10.** Comparison between the 3 methods for strategy 2. We keep the same color code as before. The best performer point for the RFE method has a less percentage of features used than the OCA method but the score of the last one is much better. Besides, the BCA method leads to a higher score than the two other methods but with a larger features set.

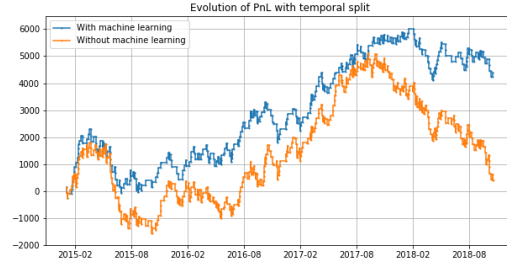
blue) curve represents our algorithmic trading strategy without any machine learning filtering (respectively with filtering done by the xgboost method trained with OCA). Since the blue curve is above the orange one, we experimentally validate that using machine learning enhances the overall profitability of our trading strategy by avoiding the bad trades.



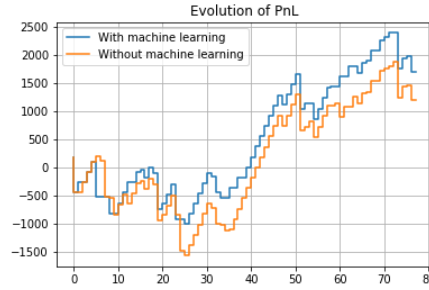
**Fig. 11.** Evolution of the PnL for strategy 1 with a randomized test set. The orange curve represents our algorithmic trading strategy without any machine learning filtering while the blue line is the result of the combination of our algorithmic trading strategy and the oca method to train our xgboost method.

If instead we split our set into two sets that are continuous in time, meaning we use as a training test the first two third of the data when there are sorted in time and as a test set the last third of the data, we get better result as the divergence between the blue and orange curve is larger. An explanation of this better efficiency may come from the fact that the non randomization of the training set makes the learning for our model easier and leads to less

overfitting overall. This method of splitting the two sets is illustrated in figures 12 and 13. Since the blue curve is above the orange one for both strategies, we experimentally validate that using machine learning enhances the overall profitability of our trading strategy by avoiding the bad trades.



**Fig. 12.** Evolution of the PnL for strategy 1 with a test set given by the last third of the data to take into account temporality in our data set. The orange curve represents our algorithmic trading strategy without any machine learning filtering while the blue line is the result of the combination of our algorithmic trading strategy and the OCA method to train our xgboost method.



**Fig. 13.** Evolution of the PnL for strategy 2 with temporal split. We keep the same color code as before.

## 6 Conclusion

In this paper, we have presented a new method, called Optimal Coordinate Ascent (OCA) that allows us selecting features among block and individual features. OCA relies on coordinate ascent to find an optimal solution for gradient

boosting methods score (number of correctly classified samples for our data). OCA takes into account the notion of dependencies between variables forming blocks in our optimization. The coordinate ascent optimization solves the issue of the NP hard original problem where the number of combinations rapidly explodes making a grid search unfeasible. It transforms the NP hard problem of finding the best features into a polynomial search one. Comparing result with two other methods, Binary Coordinate Ascent (BCA) and Recursive Feature Elimination (RFE), we find that OCA leads to the minimum feature set with the highest score. Hence, OCA provides empirically the most compact data set with optimal performance.

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